The Divergence of High- and Low-Frequency Estimation:
Implications for Performance Measurement

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Within the 14-year period ended December 31, 2013, the Equity Income Partners LP hedge fund, managed by Envision Capital, ranked in the top quartile of the CISDM universe of multistrategy funds, based on their Sharpe ratios.[1] Within the same period, the same fund ranked in the bottom quartile of the same universe, based on the same performance metric. This seeming discrepancy occurred because, in the former case, the funds' Sharpe ratios were estimated from monthly returns, whereas in the latter case they were estimated from triennial returns. The annualized numerators of the funds' Sharpe ratios did not change, but their annualized denominators did. Funds with relatively higher auto-correlated monthly returns had relatively higher standard deviations of triennial returns, thus causing them to drop in the triennial performance rankings, relative to funds with relatively lower auto-correlated monthly returns.

For funds that are evaluated based on the information ratio, which equals excess return divided by excess risk, the potential for distortion is even greater, because the information ratio is affected by the auto-correlations of the funds and their benchmarks, as well as the lagged cross-correlations between the funds and their benchmarks.

We proceed as follows. Based on earlier analysis in a companion article, we show how low-frequency standard deviations and correlations are related to their high-frequency values.[2] We then document the distortion that non-zero lagged correlations introduce to the Sharpe ratios within a universe of hedge funds, as well as to the information ratios of a universe of mutual funds. We also show how these effects distort the performance of risk parity strategies. We conclude by discussing the economic intuition of this divergence.[3]

THE MATH

Equation (1) shows the standard deviation of the cumulative continuous returns of x over q periods, $x_t + \ldots + x_{t+q-1}$, where σ_x is the standard deviation of x measured over single-period intervals. It is the denominator of the Sharpe ratio estimated over q periods.

$$\sigma(x_t + \cdots + x_{t+q-1}) = \sigma_x \sqrt{q + 2\sum_{k=1}^{q-1}(q-k)\rho_{x_t x_{t+k}}}$$

(1)

Equation 1 reveals that the standard deviation of longer-interval returns depends on the auto-correlations of the shorter-interval returns at all lags. If these auto-correlations are positive, the standard deviation of

the longer-interval returns will exceed the product of the shorter-interval returns and the square root of the number of shorter-intervals within the longer interval. In this case, the annualized Sharpe ratio will be lower when estimated from longer-interval returns than from shorter-interval returns, because the annualized mean return is invariant to the length of the estimation interval. If the auto-correlations at all lags equal zero, the standard deviation of x will scale with the square root of the horizon, q, as shown in Equation (2).

$$\sigma(x_t + \cdots + x_{t+q-1}) = \sigma_x \sqrt{q} \qquad (2)$$

Equation 3 gives the correlation between the cumulative returns of x and the cumulative returns of y over q periods.

$$\rho(x_t + \cdots + x_{t+q-1}, y_t + \cdots + y_{t+q-1})$$

$$= \frac{q\rho_{x_t y_t} + \sum_{k=1}^{q-1}(q-k)(\rho_{x_{t+k},y_t} + \rho_{x_t,y_{t+k}})}{\sqrt{q + 2\sum_{k=1}^{q-1}(q-k)\rho_{x_t,x_{t+k}}} \sqrt{q + 2\sum_{k=1}^{q-1}(q-k)\rho_{y_t,y_{t+k}}}} \quad (3)$$

The numerator equals the covariance of the assets, taking lagged cross-correlations into account, whereas the denominator equals the product of the assets' standard deviations, as specified in Equation (1).[4]

We can now use the long-horizon standard deviations of x and y, together with their correlation, to compute the expected standard deviation of their relative performance after q periods, which is called tracking error and serves as the denominator of the information ratio.

$$TE(x,y) = \sqrt{\sigma_x^2 + \sigma_y^2 - 2\rho\sigma_x\sigma_y} \qquad (4)$$

E X H I B I T   1

Percentage of Hedge Funds Within Categories that Change Quantile: Monthly vs. Triennial Sharpe Ratios

Hedge Fund Style	Number of Funds	Percentage of Funds That:							
		Change Decile		Change Quintile		Change Quartile		Cross Median	
		Result	p-Value	Result	p-Value	Result	p-Value	Result	p-Value
U.S. L/S Equity	110	51%	0%	30%	0%	35%	0%	16%	0%
International L/S Equity	57	47%	4%	33%	2%	23%	8%	14%	3%
Global Macro	32	63%	6%	47%	0%	47%	0%	9%	25%
Debt	27	70%	1%	52%	0%	22%	6%	7%	67%
Long-Only Equity	16	50%	38%	31%	6%	25%	36%	0%	n/a
FoFs/Multistrategy	209	46%	0%	26%	0%	23%	0%	11%	0%
Systematic Futures	73	45%	16%	22%	14%	25%	15%	5%	82%
Other	45	58%	1%	38%	0%	27%	0%	13%	0%
All Hedge Funds	569	50%		31%		27%		11%	

Sample includes 569 hedge funds from the CISDM/Morningstar database that report monthly returns from January 2000 through December 2013. Sharpe ratios are computed from monthly and triennial return and standard deviation, as well as the monthly and triennial return of the JP Morgan U.S. Cash Index (0.20% and 7.39%, respectively) over the same period. To increase the sample size with each category, we consolidated the CISDM/Morningstar style categories as follows: U.S. Long/Short Equity includes equity market neutral, U.S. long/short equity, and U.S. small-cap long/short equity; International Long/Short Equity includes Asia/Pacific long/short equity, emerging-markets long/short equity, Europe long/short equity, and global long/short equity; Global Macro includes global macro and currency; Debt includes debt arbitrage, distressed securities, and long/short debt; Long-Only Equity includes long-only equity and emerging-markets long-only equity; Fund-of-Funds/Multistrategy includes multistrategy funds as well as six fund-of-funds categories (macro/systematic, debt, equity, event, multistrategy, and relative value); Systematic Futures was not consolidated; Other includes bear-market equity, convertible arbitrage, event driven, long-only other, and merger arbitrage. We employ a bootstrap simulation to estimate the p-values. Specifically, we compute the frequency with which each result would arise if monthly returns were independently distributed.

We next provide empirical evidence of the extent to which lagged auto- and cross-correlations distort performance measurement.

THE DISTORTION OF HEDGE FUND PERFORMANCE

Hedge funds are typically evaluated according to their absolute performance; hence, they are compared to one another based on their Sharpe ratios. The conventional practice for evaluating hedge funds' Sharpe ratios is to estimate their standard deviations from monthly returns and then multiply these monthly standard deviations by the square root of 12. This approach implicitly assumes that monthly hedge fund returns are serially independent at all lags within a year, which is not the case. We can account for these non-zero auto-correlations by estimating their standard deviations in accordance with Equation (1), or more simply by using longer-interval returns.

Exhibit 1 reveals that the square-root-of-time shortcut significantly distorts the true annualized Sharpe ratio of longer measurement intervals. It shows the fraction of hedge funds within various categories that shift quantiles as a result of using triennial returns instead of monthly returns to compute their Sharpe ratios.

Exhibit 2 provides additional detail for U.S. Long/Short Hedge Funds. This exhibit shows how the funds that demarcate each decile based on monthly Sharpe ratios migrate to different percentile rankings, based on triennial Sharpe ratios. It reveals greater mobility among outperforming funds than among underperforming funds.

These exhibits provide compelling evidence that extrapolation of monthly Sharpe ratios to longer intervals is highly unreliable, unless one accounts for lagged correlations. Next, we explore the effect of lagged auto- and cross-correlations on mutual fund performance.

THE DISTORTION OF MUTUAL FUND PERFORMANCE

Unlike hedge funds, which are typically compared to each other according to their Sharpe ratios, mutual funds are usually evaluated by their performance relative to their given benchmarks. They are therefore compared

EXHIBIT 2
U.S. Long/Short Hedge Funds: Percentile Rank Migration
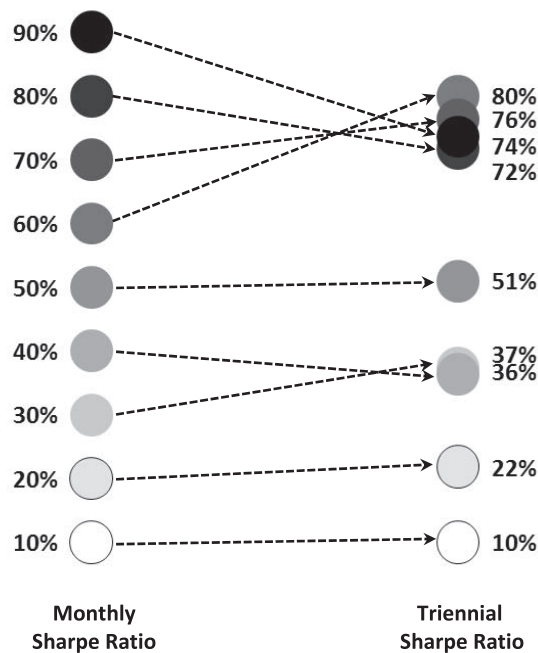
to one another based on their information ratios. The information ratio equals a fund's excess return relative to its benchmark, divided by its excess risk relative to its benchmark. The conversion of monthly information ratios to longer-interval information ratios is therefore subject to three sources of distortion: the auto-correlations of the fund's returns, the auto-correlations of the benchmark's returns, and the lagged cross-correlations between the fund's returns and the benchmark's returns. Exhibit 3 reveals that these distortions are substantial. It shows the fraction of mutual funds within various categories that changed quantiles as a consequence of shifting from monthly to triennial returns to compute their information ratios. Despite the fact that mutual fund performance is subject to more sources of distortion than is hedge fund performance, mutual funds exhibit slightly less migration than hedge funds. This result may arise because hedge fund returns tend to be more auto-correlated than are mutual fund returns.

Exhibit 3
Percentage of Mutual Funds Within Categories that Change Quantile: Monthly vs. Triennial Information Ratios

Mutual Fund Style	Number of Funds	Percentage of Funds That:							
		Change Decile		Change Quintile		Change Quartile		Cross Median	
		Result	p-Value	Result	p-Value	Result	p-Value	Result	p-Value
U.S. Large-Cap Blend	786	47%	2%	29%	0%	22%	0%	8%	0%
U.S. Small-Cap Blend	358	49%	3%	28%	2%	18%	12%	2%	99%
Foreign Lrg.-Cap Blend	392	33%	15%	15%	33%	11%	43%	2%	99%
All Mutual Funds	1536	44%		25%		18%		5%	

The U.S. Large-Cap Blend sample includes 786 out of 1,428 mutual funds listed in the "Large Blend" category on Yahoo! Finance that meet the following two criteria: monthly returns are available from January 2008 through December 2013 and tracking error relative to the S&P 500 is greater than 1% per annum. We impose the latter filter to eliminate index funds. The U.S. Small-Cap Blend sample includes 358 out of 640 funds listed in the "Small Blend" category that meet the same criteria. The Foreign Large-Cap Blend sample includes 392 funds listed in the "Foreign Large Blend" category whose monthly returns are available for the same period and whose name does not include "Idx" or "Index" (unless it also includes "Enhanced"). Monthly total returns are from Bloomberg. Returns are net of fees and assume reinvestment of dividends. The benchmarks are S&P 500, Russell 2000, and MSCI All Country World ex USA, respectively. We employ a bootstrap simulation to estimate the p-values. Specifically, we compute the frequency with which each result would arise if monthly returns were independently distributed.

THE DISTORTION OF RISK PARITY PERFORMANCE

The popular investment strategy known as risk parity is constructed so that each asset within a portfolio contributes equally to total portfolio risk, which requires that a portfolio's weights be proportional to the inverse of the assets' standard deviations. If all assets have identical Sharpe ratios and correlations, risk parity will deliver the highest Sharpe ratio of any portfolio. Of course, the Sharpe ratios and correlations of most asset classes have been substantially dissimilar to each other; hence, this result is only conceptually relevant. Nonetheless, if low-risk assets have higher Sharpe ratios than do high-risk assets, risk parity could be expected to outperform other strategies, even if Sharpe ratios and correlations are dissimilar. Indeed, risk parity has been shown to outperform a 60/40 stock and bond portfolio over a very long horizon, covering a wide variety of market conditions. It has also been shown to outperform more broadly diversified portfolios over a shorter horizon during which interest rates decline substantially, an environment which was unusually favorable to risk parity.[5]

We first compare the performance of U.S. stocks, a 60/40 stock and bond portfolio, and a risk parity portfolio from January 1929 through December 2010. The 60/40 portfolio is rebalanced monthly. To construct the risk parity portfolio, we follow the same approach described in Asness et al. [2012]. In particular, at the end of each calendar month we set the portfolio weights to be proportional to the inverse of the assets' trailing three-year monthly volatilities. Asness et al. [2012] consider two different risk parity strategies: one that is unlevered, with weights rebalanced to sum to one each month, and a second that is levered after the fact to match the volatility of a value-weighted portfolio over the full back-test history. We choose to focus exclusively on the unlevered portfolio, in part because this strategy relies only on information available at each point in time and is therefore theoretically investable. Analyzing the unlevered portfolio also lets us ignore the effect of borrowing costs, which might otherwise distract us from the issue we want to highlight. Anderson et al. [2012] investigate how market frictions affect the performance of a risk parity strategy, compared to fixed-weight and value-weighted strategies. They show that trading costs and

borrowing costs tend to reduce the Sharpe ratios of levered risk parity strategies.

Exhibit 4 shows the data we used to evaluate the performance of the risk parity strategy.

Exhibit 5 shows the annualized Sharpe ratios for risk parity, as well as the Sharpe ratios for stocks, Treasury bonds, and a 60/40 mix of stocks and bonds for a long sample beginning in 1929 and ending in 2010.[6] It clearly shows the dominance of risk parity to stocks, bonds, and a 60/40 combination thereof, based on annualization of Sharpe ratios without regard for the auto-correlation of monthly returns.

However, it also reveals that when the Sharpe ratio is computed from 10-year returns, which does account for the auto-correlation of monthly returns, the dominance of risk parity is reversed. It is now shown to underperform a 60/40 mix of stocks and bonds. In fact, risk parity underperforms the 60/40 portfolio for any measurement interval longer than 40 months. At 40 months, both strategies have a Sharpe ratio of 0.31.

Exhibit 5 also shows that the annualized volatility of 10-year returns increases much more sharply for risk parity than it does for stocks, implying that monthly risk parity returns are more positively auto-correlated than are monthly stock returns. We can quantify the effect of serial dependence directly by measuring excess dispersion, following Kinlaw et al. [2014]. Excess dispersion is the fraction of the distribution of long-horizon returns

EXHIBIT 5
Annualized Performance Statistics (Long Sample, 1929–2010)

MONTHLY HORIZON	Stocks	Bonds	60/40	Risk Parity
Return	8.90%	5.52%	8.06%	6.84%
Volatility	19.39%	7.91%	12.37%	7.62%
Sharpe Ratio	**0.27**	**0.24**	**0.36**	**0.43**
10-YEAR HORIZON	Stocks	Bonds	60/40	Risk Parity
Return	8.90%	5.52%	8.06%	6.84%
Volatility	20.88%	10.20%	12.73%	10.63%
Sharpe Ratio	**0.18**	**0.03**	**0.23**	**0.16**
EXCESS DISPERSION	Stocks	Bonds	60/40	Risk Parity
Implied, No Lags	0.00%	0.00%	0.00%	0.00%
Implied, Normal Lags	1.27%	15.95%	0.98%	18.97%
Actual	0.18%	31.51%	-1.44%	24.11%

that falls outside the one-standard-deviation tails of the distribution implied by independent and identically distributed monthly returns. The row labeled "Implied, No Lags" shows that if we ignore auto-correlations, we would estimate excess dispersion to equal zero. The row labeled "Implied, Normal Lags" shows the excess dispersion attributable solely to non-zero auto-correlations, net of non-normalities in the distribution. The row labeled "Actual" shows the empirical excess dispersion arising from all sources. The actual excess dispersion of 10-year risk parity returns is 24%. This excess dispersion likely arises from risk parity's overweight to bonds, which has excess dispersion equal to nearly 32%. It is obvious from Exhibit 5 that extrapolation of standard

EXHIBIT 4
Data Sources for Risk Parity Analysis

Long Sample (1926 –2010)		
Stocks	S&P 500 Index	From Jan 1973 - Dec 2010
	Ibbotson Stock Index	From Jan 1926 - Dec 1972
Bonds	Barclays Long Government Bond Index	From Jan 1973 - Dec 2010
	Ibbotson Treasury Bond Index	From Jan 1926 - Dec 1972
1M Risk-Free Rate	1M Treasury Yield from Thomson Datastream	From Jan 1973 - Dec 2010
	Ibbotson Treasury Bill Index	From Jan 1926 - Dec 1972
10Y Risk-Free Rate	10Y Treasury Yield from Robert Shiller's Website	From Jan 1926 - Dec 2010
Broad Sample (1973 –2010)		
Stocks	S&P 500	From Jan 1973 - Dec 2010
Treasuries	Barclays U.S. Treasury Bonds	From Jan 1973 - Dec 2010
Corporates	Barclays U.S. Corporate Bonds	From Jan 1973 - Dec 2010
Commodities	S&P GSCI Index	From Jan 1973 - Dec 2010
1M Risk-Free Rate	1M Treasury Yield from Thomson Datastream	From Jan 1973 - Dec 2010
10Y Risk-Free Rate	10Y Treasury Yield from Thomson Datastream	From Jan 1973 - Dec 2010

deviations estimated from monthly returns substantially underestimates the standard deviation of risk parity over longer horizons. In contrast, the excess dispersion of a fixed-weight 60/40 portfolio is very close to zero.

Proponents of risk parity argue that its dominance is enhanced by the fact that the security market line is relatively flat, which means that high-beta securities deliver relatively low returns, compared to low-beta securities. This asymmetry seems to be the case for the security market line of stocks and bonds estimated from their monthly returns, which has a slope of 0.59, as shown in Exhibit 6. This is not the case, however, when we estimate the security market line from 10-year returns. It approaches a slope of 1, which implies that bonds do not deliver a higher risk-adjusted return than stocks do when measured over longer intervals.

Exhibits 7 and 8 present the same analysis for a broader sample of securities: U.S. stocks, Treasury bonds, corporate bonds, and commodities. This analysis covers a shorter measurement period: 1976 through 2010. For this sample we evaluate the performance of risk parity compared to a fixed-weight portfolio and a market-weighted portfolio, both rebalanced monthly.[7] Again, the apparent superiority of risk parity disappears when the Sharpe ratio is estimated from longer-interval returns that take into account the auto-correlation of monthly returns, though risk parity's inferiority is not as significant as it was for the longer sample. But this result is to be expected. This shorter measurement period was unusually favorable to risk parity because interest rates declined precipitously throughout this period, and risk parity overweights bonds relative to other asset classes.

The bottom line is that risk parity cannot be justified by its historical performance for investors with investment horizons that extend beyond a few years. Nor does the flat security market line argument hold up when it is measured by longer-interval returns. In this broad sample, the slope of the security market line shifts from 0.46 when it is measured by monthly returns to 1.08 when it is measured by 10-year returns.

THE ECONOMIC INTUITION OF DIVERGENCE

We hypothesize that high-frequency variability arises from changes in discount rates, whereas low-frequency variability is caused by differences in cash flows. Changes in discount rates occur relatively often because a constant flow of new information causes investors to reassess the riskiness of a stream of cash flows, which therefore leads to high-frequency variability. In addition, the value of a portfolio or strategy may gradually appreciate or erode over a long horizon, because the drift of cash flows shifts upward or downward as fundamentals change. This process introduces low-frequency variability.

We test this hypothesis by analyzing the returns of 10 U.S. sectors from December 1978 through December 2013. Each month, we regress the cross-section of monthly, three-year, and ten-year sector returns on the cross-section of changes in earnings during the same periods. We also regress the cross-section of monthly, three-year, and ten-year sector returns on the cross-section of changes in beta during the same periods, where beta is computed with a three-year look-back window, relative to the S&P 500 index. We use beta as a proxy for sector discount rates, with the intuition that investors demand higher returns from sectors that have greater systematic risk.

Exhibit 9 shows the median R-squared for the cross-sectional regressions on earnings and discount rates.[8] It reveals rather starkly that high-frequency

E X H I B I T 6
Security Market Line (Long Sample, 1929–2010)
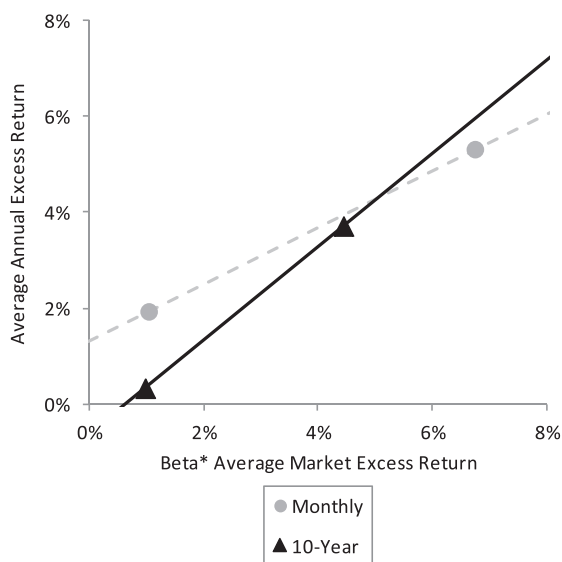
Note: The slope of the best fit line for monthly observations is 0.59. The slope of the best fit line for 10-year observations is 0.98. We use the 60/40 portfolio as a proxy for the market portfolio.
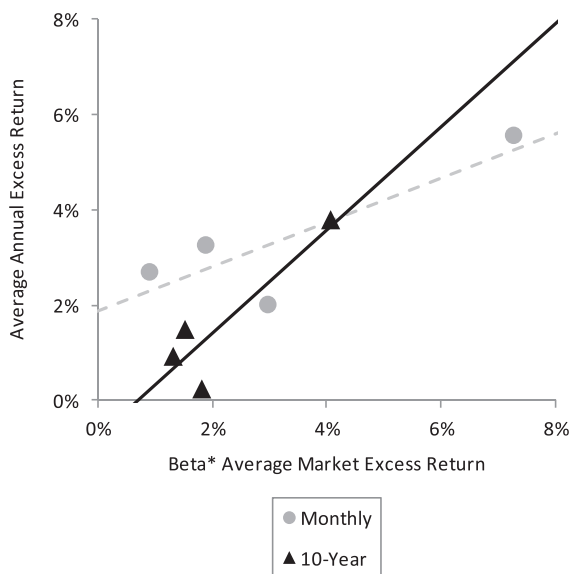
Exhibit 7
Annualized Performance Statistics (Broad Sample, 1976–2010)

MONTHLY HORIZON	Stocks	Treasuries	Corporates	Commodities	Market Weights	55/15/20/10	Risk Parity
Return	10.99%	8.12%	8.69%	7.44%	10.05%	10.22%	9.14%
Volatility	15.31%	5.41%	7.05%	19.37%	9.16%	9.60%	5.65%
Sharpe Ratio	**0.36**	**0.50**	**0.46**	**0.10**	**0.50**	**0.50**	**0.66**
10-YEAR HORIZON	Stocks	Treasuries	Corporates	Commodities	Market Weights	55/15/20/10	Risk Parity
Return	10.99%	8.12%	8.69%	7.44%	10.05%	10.22%	9.14%
Volatility	15.83%	6.22%	7.03%	11.63%	10.93%	11.02%	7.22%
Sharpe Ratio	**0.24**	**0.15**	**0.21**	**0.02**	**0.26**	**0.27**	**0.27**
EXCESS DISPERSION	Stocks	Treasuries	Corporates	Commodities	Market Weights	55/15/20/10	Risk Parity
Implied, No Lags	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Implied, Normal Lags	1.84%	9.52%	1.25%	-23.81%	9.54%	8.27%	14.22%
Actual	6.81%	23.75%	-0.17%	-23.43%	22.09%	18.77%	27.74%

Exhibit 8
Security Market Line (Broad Sample, 1976–2010)

Note: The slope of the best fit line for monthly observations is 0.46. The slope of the best fit line for 10-year observations is 1.08. We use a market portfolio constructed from annual estimates of market size.

Exhibit 9
Median R-Squared from Cross-Sectional Regression of Sector Returns

	Monthly	3-Year	10-Year
Changes in Discount Rate	0.47	0.17	0.23
Changes in Earnings	0.08	0.32	0.44

returns are more strongly related to changes in discount rates than to changes in earnings, whereas the opposite is true for relatively lower-frequency returns, which is consistent with our intuition.

CONCLUSION

Financial analysts typically extrapolate standard deviations estimated from monthly returns to longer horizons by assuming these values scale with the square root of time. Although informed analysts realize that this approach yields an approximate answer, most assume that it is a good approximation. Indeed, they are encouraged to use the square-root-of-time rule by the CFA Institute, which includes the following statement in its *Global Investment Performance Standards Handbook*, which states, "To annualize the three-year annualized (sic) ex post standard deviation calculated using monthly returns, the result of the standard deviation (S) formula above must be multiplied by the square root of 12."[9]

We respectfully disagree with this advice. First, we show mathematically how lagged auto- and cross-correlations relate high-frequency standard deviations and correlations to their low-frequency values. Next, we present empirical evidence of the extent to which naïve extrapolation of standard deviations estimated from monthly returns distorts hedge funds' three-year Sharpe ratios. We conduct a similar analysis of the effect of naïve extrapolation of tracking error on the estimation of longer-interval information ratios, based on a sample of mutual funds. We then extend these concepts to evaluate the performance of the popular investment strategy known as risk parity. Our analysis reveals that

the documented outperformance of risk parity compared to a 60/40 stock and bond portfolio is reversed when we shift from monthly estimation of standard deviations to estimates based on 10-year returns. We also show that the favorable performance of risk parity based on the monthly returns of a broader sample of asset classes vanishes when we use longer-interval returns to measure standard deviations, despite a measurement period (1976 to 2010) that was unusually favorable to bonds. Finally, we present evidence that high-frequency variability of returns is driven by changes in discount rates, whereas low-frequency variability is explained by differences in cash flows.

ENDNOTES

This article expresses the views of the authors and not State Street Global Exchange[SM].

[1]The serial correlation of the fund's monthly returns was 80%. Equity Income Partners LP ranked 3rd out of 23 funds based on monthly returns, and ranked 19th based on triennial returns. We present this example to illustrate the sensitivity of performance measurement to the frequency of estimation, not as an evaluation of the quality of this fund or any particular fund.

[2]See Kinlaw et al. [2014].

[3]Lo [2002] discusses the implications of non-zero auto-correlations on the Sharpe ratio and observes that they are highly sensitive to the return intervals used to measure volatility, based on a small sample of mutual funds and hedge funds. Lo also describes how to compute standard errors to test the significance of Sharpe ratios. We analyze a much larger sample of mutual funds and hedge funds, and we measure ranking migration net of the migration that one would expect to occur randomly. We also analyze the effect of non-zero lagged correlations on the information ratio, which requires us to analyze lagged cross-correlations as well as auto-correlations.

[4]For more detail about the mathematics of the relationship between high- and low-frequency estimation of these risk parameters, see Kinlaw et al. [2014].

[5]See, for example, Asness et al. [2012].

[6]All performance calculations are done in continuous units, so our findings are not driven by the effects of compounding on periodic returns. Average annualized excess returns are calculated as the annualized geometric mean of holding period returns, minus the annualized geometric mean of the relevant risk-free asset over the same period. Mean returns are converted to annualized units by adding 1, raising to the exponent 12 for monthly and 1/10 for 10-year,

and then subtracting 1. For a monthly horizon, the risk-free asset is a one-month U.S. Treasury bill, and for the 10-year horizon the risk-free asset is a 10-year U.S. Treasury bond. (Results are similar even if we use Treasury bills as the risk-free rate for long horizons. In this case, the Sharpe ratio of 60/40 is 0.34 and that of risk parity is 0.29.) Results for the 10-year horizon are calculated from rolling windows of 10-year (120-month) cumulative returns. To achieve equal representation of returns and ensure that performance is comparable across horizons, we splice the first 119 months onto the end of the data set before computing long-period rolling returns. Standard deviations, betas, and correlations for the 10-year horizon are calculated from the logarithm of one plus each return. To convert standard deviations to annualized units, we multiply by the square root of 12 for monthly and the square root of 1/10 for 10-year.

[7]Estimates of market capitalization weights are derived from World Bank data for equities, SIFMA data for bonds, and EIA world energy-production data for commodities. Weights are updated yearly.

[8]We ignore changes in the risk-free rate and changes in the market risk premium, because these variables are common to all sectors.

[9]*Global Investment Performance Standards Handbook, 3rd ed.,* [2012].

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