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Investors often pay managers fees that include a base component and a variable component, especially managers of hedge funds and private equity. These fee arrangements are called *performance fees*. Alternatively, investors pay some managers a fee amount that is a fixed percentage of assets under management. Investors must, therefore, determine which fee arrangement will yield the better after-fee performance. The answer to this question is anything but apparent because there are so many variables that affect after-fee performance in ways that are both complex and subtle.

Let's consider the following example. An investor is offered the choice of two fee arrangements: a flat fee that pays 1.0% of assets under management or a performance fee that includes a base fee of 0.5% of assets under management and a performance fee equal to 20% of the gains in excess of the base fee. Moreover, the performance component of this fee is subject to a high water mark, in which the manager is only paid the performance component if the cumulative return in excess of the benchmark is above the cumulative base fee. This choice is offered by five managers, each of whom is expected to deliver a return above the benchmark equal to 3.0% with tracking error equal to 6.0%. Also, their returns are 50% correlated with each other.

If we were to consider only the expected return net of fees, we would prefer the flat fee because, based on a five-year simulation of performance, it produces an expected after-fee return equal to 2.04% compared to 1.87% for the performance fee. However, if we were to consider the entire distribution of expected after-fee returns, given a particular utility function,[1] we would select the performance fee because it yields a certainty equivalent equal to 0.50% compared to 0.31% for the flat fee. This is the main contribution of our article. We evaluate after-fee performance based on the entire range of potential outcomes, taking into account the preferences of the investor.

METHODOLOGY

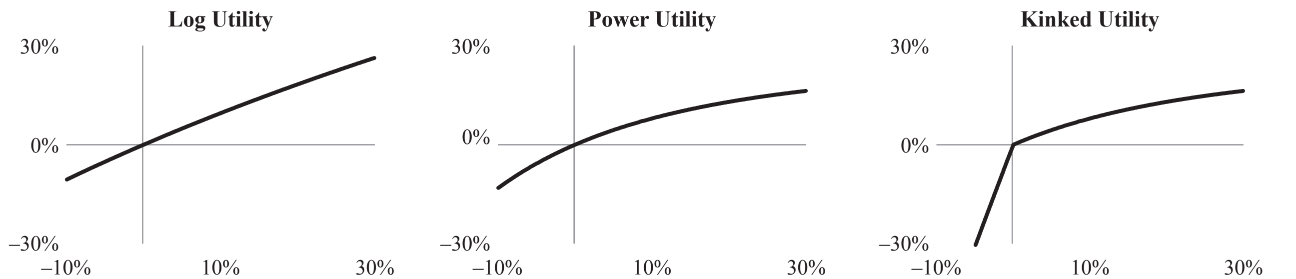
We consider three determinants of after-fee performance: the structure of the fees, the expected performance of the managers, and the preferences of the investor. We start with the following base case.

Fee Structure

1. Flat fee: 1% of assets under management
2. Performance fee: (1) base fee equal to 0.5% of assets under management, (2) performance component equal to 20%

[1] This example assumes that the investor has a kinked utility function that equals log-wealth utility plus an additional penalty of 5 per unit of negative return.

EXHIBIT 1
Utility Functions



of gains in excess of base fee, and (3) high water mark

Manager Information
Five managers:

1. Expected return in excess of benchmark for each manager equal to 3.0%
2. Expected tracking error for each manager equal to 6.0%
3. Correlation equal to 0.0%
4. Managers retained for entire five-year measurement period

Investor Preferences
We express investor preferences as a function of returns:

1. Log utility: $U_{log}(R) = \ln(1 + R)$
2. Power utility: $U_{power}(R) = \dfrac{(1+R)^{1-\gamma} - 1}{1-\gamma}$
3. Kinked utility:

$$U_{kinked}(R) = \begin{cases} \dfrac{(1+R)^{1-\gamma} - 1}{1-\gamma}, & R \geq \theta \\ \lambda(R-\theta) + \dfrac{(1+R)^{1-\gamma} - 1}{1-\gamma}, & R < \theta \end{cases}$$

In these equations, U equals utility, R equals return, γ is a risk aversion coefficient, θ is the location of the kink, and λ is a linear penalty coefficient.

Exhibit 1 depicts these utility functions for returns ranging from –10% to 30%. For power utility, we set the risk aversion coefficient γ, equal to 5. Kinked utility is equal to power utility with γ equal to 5, plus an additional penalty for returns below the kink threshold. We set the kink threshold, θ equal to 0% and the penalty coefficient, λ equal to 5. Relative to log utility, both the power utility function and kinked utility function represent increased aversion to risk. Power utility increases risk aversion by increasing the degree of curvature in the utility function. Kinked utility further increases risk aversion by imposing an asymmetric penalty on returns below zero.

Based on these fee structures, information about the managers, and investor preferences, we simulate 10,000 five-year paths of a portfolio's cumulative return net of fees and calculate the certainty equivalents of the distributions of after-fee returns. These certainty equivalents determine which fee is preferable for a particular investor.

We perform these simulations for the base case and for a variety of situations in which we vary the assumptions that define each case. These variations allow us to gauge the sensitivity of the preferred fee to the structure of the fee, the performance of the managers, and the preferences of the investor.

Throughout this article, we use the term *return* to refer to a fund's return in excess of the benchmark, and we use the terms *standard deviation* and *volatility* to refer to the standard deviation of the fund's returns in excess of the benchmark.

CERTAINTY EQUIVALENTS

Before we proceed, it may be useful to review the notion of a *certainty equivalent*. This is the value of a certain prospect that yields the same utility as the expected utility of an uncertain prospect. This notion was introduced as far back as 1738 by the famous mathematician Daniel Bernoulli [1954] to determine the amount one should be willing to pay to insure an item of value that was being shipped across the ocean. Kritzman [2004] employed certainty equivalents to compare portfolios produced by mean–variance analysis to portfolios determined by full-scale optimization. Later, Goetzmann et al. [2007] showed how to gauge active performance using certainty equivalents.

Here is a simple example of a certainty equivalent. Consider an investor who has log-wealth utility and is faced with a risky investment that has an equal probability of increasing by one-third or falling by one-quarter. The utility of this investment equals the sum of the probability-weighted utilities of the two outcomes. If the initial investment is \$100.00, the expected utility of this investment equals 4.60517 [$= \ln(133.33) \times 0.50 + \ln(75.00) \times 0.50$].

This investment has an expected value of \$104.17, but the actual return that will prevail is uncertain. How much less should the investor be willing to accept with certainty such that the investor would be indifferent between this amount and an uncertain value of \$104.17? It turns out that \$100.00 also yields utility of 4.6052 [$\ln(100) = 4.60517$]. Therefore, if the investor has log-wealth utility, the investor would be indifferent between receiving \$100.00 for certain and an equal probability of receiving \$133.33 or \$75.00.

For a log-wealth utility function, we find the certainty equivalent by raising *e*, the base of the natural logarithm, to the power of expected utility: $100.00 = e^{\ln(133.33)\times 0.50+\ln(75.00)\times 0.50.}$ In our application of certainty equivalents and assuming log-wealth utility, rather than raising *e* to the probability-weighted sum of the natural logarithms to just two outcomes, we raise *e* to the probability-weighted sum of the natural logarithms of the 10,000 possible outcomes generated by our simulation. Similarly, for the other utility functions, we solve for the value that has the same utility as the probability-weighted sum of the utilities associated with the 10,000 simulated outcomes. Again, in our simulations the outcomes are defined as after-fee portfolio returns. In our previous example, utility is defined as a function of wealth.

RESULTS

We begin by comparing the certainty equivalent of a flat fee of 1% with that of a performance fee with a base component of 0.5% and a performance component equal to 20% of gains in excess of the base fee that is subject to a high water mark. Our initial analysis assumes the investor has kinked utility, as defined previously. We also assume there are five managers with uncorrelated returns, each of which has an expected value of 3% and a standard deviation of 6%. Finally, we assume that no manager will be terminated during the five-year investment horizon. We present our results in certainty equivalent units, which, as we discussed earlier, are based on 10,000 simulated outcomes. Unlike the mean after-fee return and standard deviation, or the option value of the fee, these certainty equivalents take into account all of the features of the fees, the managers' performance, and the investor's preferences.

EXHIBIT 2
Flat Fee versus Performance Fee (certainty equivalents assuming kinked utility)

	Flat Fee	Performance Fee
Returns	2.80%	2.80%
Fees	–1.08%	–1.31%
Net Returns	1.65%	1.59%

Exhibit 2 requires some explanation. First of all, returns, fees, and net returns are in certainty equivalent units. These certainty equivalents pertain to the returns, fees, and net returns in isolation. They are not additive because the utility function penalizes losses more than it rewards gains. For example, a large fee by itself would convey a large amount of disutility. In the case of a performance fee, however, a large fee would be associated with a large return, which conveys large positive utility. Therefore, because larger fees are associated with larger returns, the certainty equivalent of net returns will not change linearly with changes in the certainty equivalent of fees. It is the certainty equivalent of net returns that determines which fee is preferable to the investor.

Next, in Exhibit 3 we show how these values vary as we change our assumptions about the volatility of the managers' returns, the correlation of their returns, and the number of managers. We also allow for manager termination based on a range of return thresholds. In the simulations, we terminate any manager whose two-year cumulative performance falls below the termination threshold, and we reinvest this portion of the portfolio in a new contract with a new manager, which resets the

EXHIBIT 3

Performance Fee Sensitivity (certainty equivalents assuming kinked utility)

Volatility	**3%**	**6%**	**9%**	**12%**	**15%**	**18%**
Returns	2.94%	2.80%	2.30%	1.25%	0.13%	–0.22%
Fees	–1.15%	–1.31%	–1.48%	–1.67%	–1.84%	–2.04%
Net Returns	1.94%	1.59%	0.62%	–0.12%	–0.39%	–0.75%
Correlation	**–20%**	**0%**	**20%**	**40%**	**60%**	**80%**
Returns	2.96%	2.80%	2.50%	2.08%	1.59%	1.25%
Fees	–1.31%	–1.31%	–1.31%	–1.31%	–1.30%	–1.31%
Net Returns	1.81%	1.59%	1.16%	0.68%	0.20%	–0.02%
Number of Managers	**1**	**5**	**10**	**20**		
Returns	0.92%	2.80%	2.90%	2.95%		
Fees	–1.33%	–1.31%	–1.31%	–1.31%		
Net Returns	–0.06%	1.59%	1.76%	1.81%		
Termination Threshold	**0%**	**–5%**	**–10%**	**–20%**	**None**	
Returns	2.82%	2.81%	2.79%	2.80%	2.80%	
Fees	–1.34%	–1.32%	–1.31%	–1.31%	–1.31%	
Net Returns	1.57%	1.57%	1.58%	1.59%	1.59%	

high water mark. The values in the exhibit pertain to the investor with kinked utility as described earlier. In Appendix A, we reproduce these results for investors with log utility and power utility. In each panel, we retain the base case assumptions for the other parameters, and we shade the values that are associated with our base case assumptions.

Exhibit 3 reveals that the certainty equivalent of net returns decreases with volatility and correlation, rises with the number of managers, and rises only modestly as the termination threshold becomes less stringent. It decreases with increases in volatility because a higher standard deviation increases the likelihood of an extreme return, and a kinked utility function penalizes an extreme negative return more than it rewards an extreme positive return of the same magnitude. The certainty equivalent of net returns falls as correlations rise because the portfolio of managers becomes less diversified and therefore more volatile. It rises with the number of managers for the opposite reason. Finally, the certainty equivalent of net return rises slightly as it becomes less likely that a manager will be terminated; this occurs because retained managers, unlike terminated managers, have the potential to recoup losses that accrue to the investor as a result of the high water mark provision.

Thus far, we have presented results for an investor with kinked utility. Exhibit 4 shows the sensitivity of net returns to volatility, correlation, number of managers, and termination threshold for investors with log utility and power utility, as well as for investors with kinked utility.

Exhibit 4 reveals why the option value of a performance fee is inadequate for assessing the desirability of a particular fee structure. Although the option value of a particular fee structure is invariant to investor preferences, Exhibit 4 clearly shows that investors with different utility functions value the same fee structure differently. The key insight from these exhibits is that an investor's preference for a particular fee is a highly complex function of the structure of the fee, the expected performance of the managers, and the investor's attitude toward risk.

Exhibits 3 and 4 pertain only to performance fees. In Exhibits 5, 6, and 7, we provide a direct comparison of performance fees and flat fees. For a given set of assumptions, an investor would choose the fee structure that provides the highest certainty equivalent of net returns. In the event that these two certainty equivalents are not equal, we calculate the incremental return—which, when added to the expected returns of the managers with performance fees, would render the investor indifferent between that portfolio and a

Exhibit 4

Performance Fee Sensitivity for Various Utility Functions (certainty equivalents of net returns)

Volatility	3%	6%	9%	12%	15%	18%
Log Utility	1.96%	1.82%	1.63%	1.44%	1.15%	0.89%
Power Utility	1.94%	1.72%	1.38%	0.95%	0.41%	–0.26%
Kinked Utility	1.94%	1.59%	0.62%	–0.12%	–0.39%	–0.75%
Correlation	**–20%**	**0%**	**20%**	**40%**	**60%**	**80%**
Log Utility	1.84%	1.82%	1.80%	1.76%	1.71%	1.73%
Power Utility	1.81%	1.72%	1.60%	1.48%	1.37%	1.27%
Kinked Utility	1.81%	1.59%	1.16%	0.68%	0.20%	–0.02%
Number of Managers	**1**	**5**	**10**	**20**		
Log Utility	1.70%	1.82%	1.82%	1.83%		
Power Utility	1.15%	1.72%	1.77%	1.81%		
Kinked Utility	–0.06%	1.59%	1.76%	1.81%		
Termination Threshold	**0%**	**–5%**	**–10%**	**–20%**	**None**	
Log Utility	1.78%	1.81%	1.80%	1.82%	1.82%	
Power Utility	1.69%	1.70%	1.71%	1.72%	1.72%	
Kinked Utility	1.57%	1.57%	1.58%	1.59%	1.59%	

Exhibit 5

Breakeven Return for Investor with Kinked Utility

Flat Fee	1.65%
Performance Fee	1.59%
Breakeven Return	0.05%

portfolio of managers that charge a flat fee. We call this the *breakeven return*. Exhibit 5 presents this result for an investor with kinked utility, using the base case assumptions described earlier.

Exhibit 5 reveals that the net returns resulting from performance fees have a lower certainty equivalent than the net returns that result from a flat fee. In particular, the flat fee structure is 0.05% more valuable to the investor than the performance fee structure. For an investor to be indifferent between these two options, the portfolio with performance fees would need to generate 0.05% more value, which could be achieved through any combination of a reduction in fees, an increase in expected return, or a decrease in risk. The breakeven return in Exhibit 5 represents the size of the increase in expected return that would generate the required value.[2]

Exhibit 6 shows how the breakeven return differs given different assumptions about volatility, correlation, number of managers, and termination threshold, again for an investor with kinked utility. In Appendix A, we reproduce these results for investors with log utility and power utility.

Exhibit 6 shows that as volatility increases, the incremental return required for an investor to be indifferent between a performance fee and a flat fee increases. This result occurs because with greater volatility, there is a greater likelihood of one manager performing well and demanding a fee while all other managers perform poorly. This scenario results in large disutility in the case of performance fees because the fee paid to the outperforming manager causes a further decline to a portfolio return that is already negative. In contrast, the fee paid to the outperforming manager would be capped in the case of flat fees.

As correlation increases, we see the opposite relationship. When correlations are high, there is less dispersion in manager performance. Therefore, individual manager returns—and performance fees—tend to align with total portfolio returns. In this scenario, investors are unlikely to pay performance fees when the total portfolio return is negative. As a result, performance fees are more desirable than flat fees, which investors incur regardless of portfolio performance.

[2] Because of the nonlinearity of the utility function, the breakeven return may not exactly equal the difference in certainty equivalents.

EXHIBIT 6
Breakeven Sensitivity Analysis for Investor with Kinked Utility

Volatility	**3%**	**6%**	**9%**	**12%**	**15%**	**18%**
Flat Fee	1.94%	1.65%	0.79%	–0.06%	–0.28%	–0.58%
Performance Fee	1.94%	1.59%	0.62%	–0.12%	–0.39%	–0.75%
Breakeven Return	0.01%	0.05%	0.12%	0.21%	0.32%	0.43%
Correlation	**–20%**	**0%**	**20%**	**40%**	**60%**	**80%**
Flat Fee	1.96%	1.65%	1.10%	0.52%	–0.01%	–0.07%
Performance Fee	1.81%	1.59%	1.16%	0.68%	0.20%	–0.02%
Breakeven Return	0.17%	0.05%	–0.04%	–0.11%	–0.16%	–0.19%
Number of Managers	**1**	**5**	**10**	**20**		
Flat Fee	–0.13%	1.65%	1.89%	1.95%		
Performance Fee	–0.06%	1.59%	1.76%	1.81%		
Breakeven Return	–0.22%	0.05%	0.14%	0.17%		
Termination Threshold	**0%**	**–5%**	**–10%**	**–20%**	**None**	
Flat Fee	1.67%	1.65%	1.64%	1.65%	1.65%	
Performance Fee	1.57%	1.57%	1.58%	1.59%	1.59%	
Breakeven Return	0.09%	0.07%	0.06%	0.06%	0.05%	

EXHIBIT 7
Breakeven Analysis for Investors with Different Utility Functions

Volatility	**3%**	**6%**	**9%**	**12%**	**15%**	**18%**
Log Utility	0.02%	0.17%	0.33%	0.50%	0.64%	0.80%
Power Utility	0.01%	0.13%	0.25%	0.35%	0.44%	0.51%
Kinked Utility	0.01%	0.05%	0.12%	0.21%	0.32%	0.43%
Correlation	**–20%**	**0%**	**20%**	**40%**	**60%**	**80%**
Log Utility	0.18%	0.17%	0.16%	0.15%	0.15%	0.14%
Power Utility	0.17%	0.13%	0.09%	0.05%	0.01%	–0.02%
Kinked Utility	0.17%	0.05%	–0.04%	–0.11%	–0.16%	–0.19%
Number of Managers	**1**	**5**	**10**	**20**		
Log Utility	0.14%	0.17%	0.17%	0.18%		
Power Utility	–0.06%	0.13%	0.15%	0.17%		
Kinked Utility	–0.22%	0.05%	0.14%	0.17%		
Termination Threshold	**0%**	**–5%**	**–10%**	**–20%**	**None**	
Log Utility	0.20%	0.19%	0.17%	0.17%	0.17%	
Power Utility	0.16%	0.14%	0.13%	0.13%	0.13%	
Kinked Utility	0.09%	0.07%	0.06%	0.06%	0.05%	

Exhibit 7 shows how these sensitivities vary across investors with different utility functions. The exhibit shows that investors with log utility require higher expected returns than investors with power or kinked utility to rationalize a performance fee. It also shows that the breakeven return is more sensitive to volatility for an investor with log utility than for investors with power utility or kinked utility. Furthermore, the exhibit reveals that investors with kinked utility are more sensitive than investors with power utility or log utility to changes in correlation and the number of managers. The impact of the termination threshold is similar for investors with any of the three utility functions.

The results we present in Exhibits 2 through 7 are specific to the particular utility functions we assumed and the parameters that we assigned to these utility functions. If we were to choose a different utility function or change the parameters for the utility functions we have evaluated, the results would certainly be quantitatively different and perhaps qualitatively different as well. The key takeaway from this analysis is that interaction among the structure of a performance fee, the expected performance of the managers, and the preferences of the investor is extremely complex.

THE EFFECT OF PERFORMANCE FEES ON VOLATILITY AND EXPECTED RETURN

Up to this point, we have focused on results derived from the entire distribution of returns net of fees. We have shown that the asymmetric nature of performance fees causes this distribution to be nonnormal; therefore, it cannot be described entirely by its mean and standard deviation. Nevertheless, for some applications such as portfolio optimization, it may be useful to summarize the distribution of net returns in terms of mean and standard deviation. The asymmetric impact of performance fees can bias these two statistics.

The observed volatility of returns net of fees understates the risk associated with a fund that charges performance fees. This occurs for two reasons. First, the performance fee lowers the mean return around which the deviations are estimated, which reduces the apparent size of downside deviations.[3] Second, the performance component of the fee reduces upside deviations because part of the upside return—but not downside deviations—is transferred to the managers. However, the calculation of standard deviation does not distinguish between upside and downside deviations. Hence, performance fees reduce after-fee standard deviations but not downside deviations. Exhibit 8 shows how the observed standard deviation understates the true risk of a fund that charges performance fees. These values are based on our base case assumptions for performance fees.

Expected returns may also be distorted by performance fees. It is tempting to conclude that an investor who allocates to a group of managers who each charge 20% performance fees should expect to pay 20% of the portfolio's total return in fees, but this assumption is incorrect. The average of multiple performance fees applied to individual fund returns will always be greater than the performance fee applied to the average fund return. Therefore, subtracting the performance fee associated with a portfolio's composite returns will overestimate the true expected return net of fees when there are multiple managers who charge performance fees. Exhibit 9 shows the extent to which portfolio-level fee calculations will overstate expected net returns. The overstatement is largest when fund volatility and number of managers are large and correlations are low. These characteristics increase the likelihood that performance will diverge substantially across managers, in which case the underperforming managers will drag down portfolio returns but not reimburse the large fees paid to the outperforming managers.

EXHIBIT 8
Understatement of Risk

Observed Standard Deviation	Mean Correction	Mean + Upside Deviation Correction
2.22%	2.38%	2.53%

CONCLUSION

Investors are often faced with a choice between a flat fee and a performance fee. If they choose a performance fee, they must decide which particular features of the performance fee will best serve them. Unfortunately, the best choice depends on a complex set of interactions among the structure of the fee, the expected performance of the managers, and the preferences of the investor. The expected cost of the fee is inadequate for determining the appropriate fee because, among other reasons, it fails to account appropriately for the expected performance of the manager. A manager who charges a high fee, for example, may be justified in doing so if expected to deliver a sufficiently high return. Therefore, at a minimum we must focus on after-fee performance rather than simply on the expected cost of the fee. However, the expected value and dispersion of after-fee performance are also inadequate determinants of the appropriate fee structure because performance fees result in nonlinear payoffs. This might lead us to

[3] This can be shown mathematically. We provide the mathematical proof in the online supplement.

Exhibit 9
Overstatement of Expected Return

Volatility	3%	6%	9%	12%	15%	18%
Average Reduction	0.06%	0.22%	0.36%	0.50%	0.64%	0.78%
Correlation	**–20%**	**0%**	**20%**	**40%**	**60%**	**80%**
Average Reduction	0.27%	0.22%	0.17%	0.12%	0.08%	0.04%
Number of Managers	**1**	**5**	**10**	**20**		
Average Reduction	0.00%	0.22%	0.25%	0.27%		

consider the option value of a performance fee as the appropriate metric for evaluating performance fees, but this measure is also inadequate because it is invariant to investor preferences. We argue that the appropriate approach for evaluating performance fees and comparing them to an alternative flat fee is to simulate the distribution of after-fee returns and, given a particular utility function, to use the certainty equivalent of the after-fee return distribution to determine the appropriate fee structure.

Although there are no simple rules to guide investors in their choice of a fee structure, we believe that our certainty equivalent methodology will enable investors to determine the optimal fee structure for a given set of circumstances.

Appendix

Exhibit A1 presents base case results and sensitivity results for an investor with log utility.

Exhibit A2 presents breakeven returns for an investor with log utility.

Exhibit A3 presents base case results and sensitivity results for an investor with power utility with γ equal to 5.

Exhibit A4 presents breakeven returns for an investor with power utility with γ equal to 5.

Exhibit A1
Performance Fee Sensitivity (certainty equivalents assuming log utility)

	Flat Fee	Performance Fee				
Returns	2.98%	2.97%				
Fees	–1.08%	–1.31%				
Net Returns	1.98%	1.82%				
Volatility	**3%**	**6%**	**9%**	**12%**	**15%**	**18%**
Returns	2.98%	2.97%	2.92%	2.88%	2.73%	2.62%
Fees	–1.15%	–1.31%	–1.48%	–1.67%	–1.84%	–2.04%
Net Returns	1.96%	1.82%	1.63%	1.44%	1.15%	0.89%
Correlation	**–20%**	**0%**	**20%**	**40%**	**60%**	**80%**
Returns	2.99%	2.97%	2.94%	2.89%	2.83%	2.86%
Fees	–1.30%	–1.31%	–1.31%	–1.31%	–1.31%	–1.32%
Net Returns	1.84%	1.82%	1.80%	1.76%	1.71%	1.73%
Number of Managers	**1**	**5**	**10**	**20**		
Returns	2.81%	2.97%	2.98%	2.99%		
Fees	–1.31%	–1.31%	–1.31%	–1.31%		
Net Returns	1.70%	1.82%	1.82%	1.83%		
Termination Threshold	**0%**	**–5%**	**–10%**	**–20%**	**None**	
Returns	2.96%	2.97%	2.95%	2.97%	2.97%	
Fees	–1.34%	–1.32%	–1.31%	–1.31%	–1.31%	
Net Returns	1.78%	1.81%	1.80%	1.82%	1.82%	

EXHIBIT A2

Breakeven Sensitivity Analysis for Investor with Log Utility

Flat Fee	1.97%					
Performance Fee	1.82%					
Breakeven Return	0.17%					
Volatility	**3%**	**6%**	**9%**	**12%**	**15%**	**18%**
Flat Fee	1.98%	1.97%	1.92%	1.88%	1.72%	1.61%
Performance Fee	1.96%	1.82%	1.63%	1.44%	1.15%	0.89%
Breakeven Return	0.02%	0.17%	0.33%	0.50%	0.64%	0.80%
Correlation	**–20%**	**0%**	**20%**	**40%**	**60%**	**80%**
Flat Fee	1.99%	1.97%	1.94%	1.89%	1.91%	1.85%
Performance Fee	1.84%	1.82%	1.80%	1.76%	1.78%	1.73%
Breakeven Return	0.18%	0.17%	0.16%	0.15%	0.15%	0.14%
Number of Managers	**1**	**5**	**10**	**20**		
Flat Fee	1.88%	1.97%	1.98%	1.99%		
Performance Fee	1.76%	1.82%	1.82%	1.83%		
Breakeven Return	0.14%	0.17%	0.17%	0.18%		
Termination Threshold	**0%**	**–5%**	**–10%**	**–20%**	**None**	
Flat Fee	1.97%	1.97%	1.95%	1.97%	1.97%	
Performance Fee	1.79%	1.81%	1.80%	1.82%	1.82%	
Breakeven Return	0.20%	0.19%	0.17%	0.17%	0.17%	

EXHIBIT A3

Performance Fee Sensitivity (certainty equivalents assuming power utility)

	Flat Fee	**Performance Fee**				
Returns	2.83%	2.83%				
Fees	–1.13%	–1.32%				
Net Returns	1.83%	1.72%				
Volatility	**3%**	**6%**	**9%**	**12%**	**15%**	**18%**
Returns	2.94%	2.83%	2.60%	2.28%	1.83%	1.22%
Fees	–1.15%	–1.32%	–1.50%	–1.70%	–1.88%	–2.10%
Net Returns	1.94%	1.72%	1.38%	0.95%	0.41%	–0.26%
Correlation	**–20%**	**0%**	**20%**	**40%**	**60%**	**80%**
Returns	2.96%	2.83%	2.68%	2.53%	2.39%	2.26%
Fees	–1.31%	–1.32%	–1.32%	–1.33%	–1.33%	–1.34%
Net Returns	1.81%	1.72%	1.60%	1.48%	1.37%	1.27%
Number of Managers	**1**	**5**	**10**	**20**		
Returns	2.11%	2.83%	2.90%	2.96%		
Fees	–1.35%	–1.32%	–1.31%	–1.31%		
Net Returns	1.15%	1.72%	1.77%	1.81%		
Termination Threshold	**0%**	**–5%**	**–10%**	**–20%**	**None**	
Returns	2.83%	2.83%	2.83%	2.83%	2.83%	
Fees	–1.35%	–1.33%	–1.32%	–1.32%	–1.32%	
Net Returns	1.69%	1.70%	1.71%	1.72%	1.72%	

EXHIBIT A4
Breakeven Sensitivity Analysis for Investor with Power Utility

Flat Fee	1.83%					
Performance Fee	1.72%					
Breakeven Return	0.13%					
Volatility	**3%**	**6%**	**9%**	**12%**	**15%**	**18%**
Flat Fee	1.94%	1.83%	1.60%	1.27%	0.81%	0.20%
Performance Fee	1.94%	1.72%	1.38%	0.95%	0.41%	–0.26%
Breakeven Return	0.01%	0.13%	0.25%	0.35%	0.44%	0.51%
Correlation	**–20%**	**0%**	**20%**	**40%**	**60%**	**80%**
Flat Fee	1.96%	1.83%	1.68%	1.52%	1.38%	1.26%
Performance Fee	1.81%	1.72%	1.60%	1.48%	1.37%	1.27%
Breakeven Return	0.17%	0.13%	0.09%	0.05%	0.01%	–0.02%
Number of Managers	**1**	**5**	**10**	**20**		
Flat Fee	1.10%	1.83%	1.90%	1.96%		
Performance Fee	1.15%	1.72%	1.77%	1.81%		
Breakeven Return	–0.06%	0.13%	0.15%	0.17%		
Termination Threshold	**0%**	**–5%**	**–10%**	**–20%**	**None**	
Flat Fee	1.83%	1.82%	1.83%	1.83%	1.83%	
Performance Fee	1.69%	1.70%	1.71%	1.72%	1.72%	
Breakeven Return	0.16%	0.14%	0.13%	0.13%	0.13%	

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